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Energy Gain Process of a Celestial Body

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Abstract
The article considered in this paper attempts to explain the astrophysical phenomena of 'dark energy' and 'dark matter' as curvature effects in a modified theory of gravity. The deviations of this theory from Einstein's general relativity are not expected to be observed on Solar System scales, but are relevant on galactic or higher scales. These properties allow the theory to survive Solar System tests of general relativity that currently constrain such models (for instance, [1] finds that GR holds in the Solar System to within 0.5%), but still permit it to provide an alternative explanation of dark matter and dark energy. In order to understand the proposed explanation however, one must first review what cosmologists mean by dark matter and dark energy, why they are largely required in the standard cosmological model, and what kind of observational evidence would an alternative model have to match.
Keywords: Astrophysics, cosmology, mass, gravity and gravitational.

I. Introduction

As the name implies, dark matter acts like regular matter gravitationally, but does not emit any EM radiation that can be observed on Earth. Dark matter is the widely accepted explanation for a large number of anomalies observed in galaxies. These anomalies occur when the total mass is calculated by different methods, and the results strongly disagree. The total mass of a galaxy, as well as its distribution, can be easily computed from the velocity distribution of the observed components, via the virial theorem. This calculation can be done classically, since GR corrections are negligible for the distances involved. As early as 1933, observations of galactic clusters showed that the speeds at which some components were seen to orbit the center were much higher than the mass estimate would allow – in fact, for some estimates the amount of mass inside the cluster would have needed to be 400 times greater than inferred from the amount of visible matter. This became known as the "missing mass" problem. Further to the missing mass problem is the problem of rotation curves. Rotation curves indicate the orbiting velocity of stars or dust around the center of the galaxy. The concept can in principle apply to any gravitationally bound system, such as the Solar System or galaxy clusters, but the problem was first seen in the study of spiral galaxies.

According to Kepler's third law, rotation curves must approach zero as one nears the edge of such a galaxy. Observationally, however, the rotation curves are largely flat outside the center. (Figure 1.)

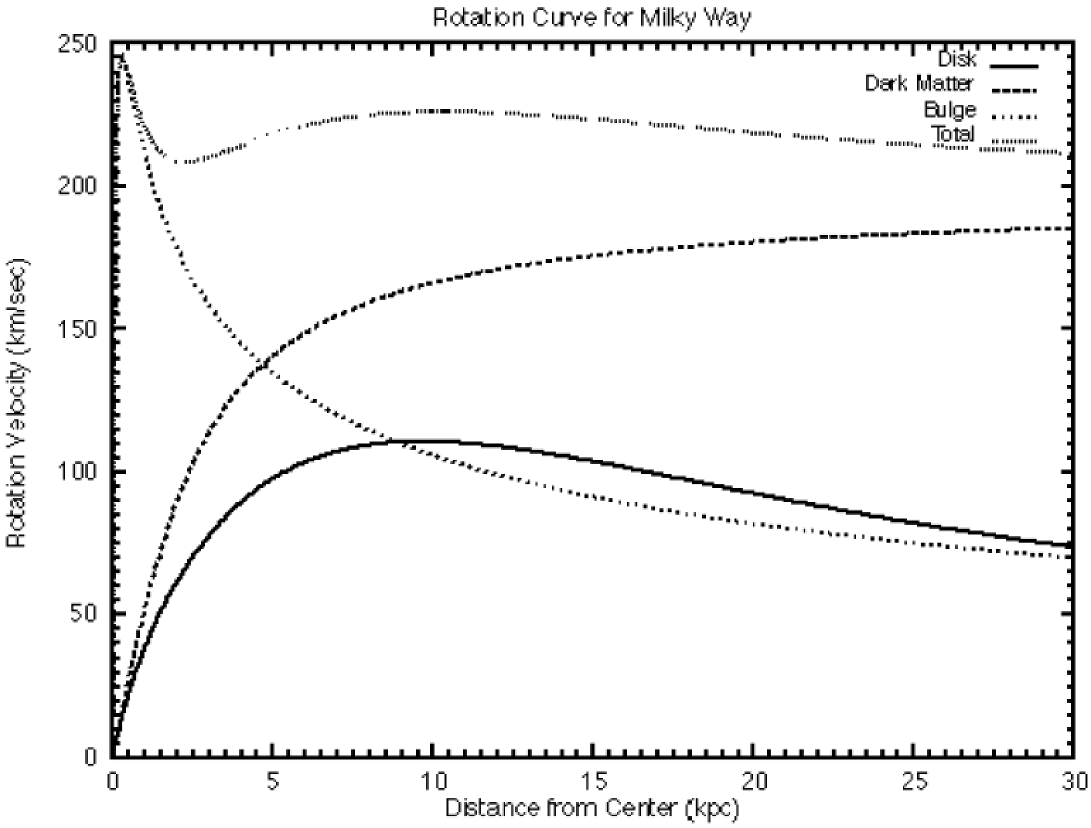

Figure 1. Rotation curve for the Milky Way, including individual contributions. Source: [2]

The galaxy rotation problem can be easily solved assuming that the galaxy contains a large quantity of dark matter, since its distribution can be selected to match any rotation curve. Unfortunately, this means that dark matter becomes a re-wrapping of our own ignorance. Very much like the original postulation of the neutrino to conserve energy and momentum in beta decays, dark matter would be simply a book-keeping device - one can infer nothing about it other than its distribution and the fact that it acts gravitationally like regular matter. Still, the experimental fact remains that a majority of galaxies' mass as inferred from rotation curves seems to consist of dark matter.

Dark matter also plays an important role in the formation of structure in the early universe. The structure of the universe that we observe–galaxies, stars, and other largescale objects–evolved from small fluctuations in the plasma of the early universe that underwent gravitational collapse over the eons. Without dark matter, structure can only be formed by ordinary baryonic matter. But up to the recombination era, ordinary matter is coupled to the photons in the universe. This coupling results in a restoring force that acts to prevent further collapse; the result is acoustic oscillations and inhibition of structure formation. Such a picture would not be able to produce the amount of structure that is observed. The addition of dark matter (assuming it is still 'dark' at those energies, i.e. it is decoupled from the photons) changes the picture since dark matter is free to collapse gravitationally without resulting in a restoring force. This helps the formation of structure around local concentrations of dark matter. Current results from the WMAP experiment support the existence of dark matter in the early universe in amounts comparable to those today, indicating that dark matter is a long-lived species.

Little can be said about the nature of dark matter itself. Dark matter can be either relativistic or non-relativistic. In fact some relativistic (hot) dark matter is already known to exist: neutrinos, since they have been confirmed to be massive by the K2K and SNO experiments. However, they cannot account for a large proportion of the dark matter content, given that their masses and number densities are fairly well known. Nor can hot dark matter account for smaller scales of structure formation, simply because it moves too fast. The picture most consistent with the experimental data is that the dark matter is (and was for most of the universe's history) cold (non-relativistic). The most likely explanation is that it is some sort of massive, very weakly-interacting particle. GUTs can provide a number of candidates for dark matter; for instance, the lightest supersymmetric particle. Such a particle would have a mass on the GUT scale, decouples from matter at that energy scale, and has a very long lifetime – thus being a good DM candidate.

Unfortunately, until a leading GUT emerges, it is largely impossible to make predictions about the interactions of the cold dark matter assumed to be present in the universe.

1.2 Energy Process

Dark energy refers to a form of energy that has negative pressure. More specifically, it has an equation of state ρ =wp with *w* < -1/3. It was conceived by Einstein, who wrote the equation for the metric in order to accommodate a static universe

$$R_{\mu v} - \frac{1}{2} R g_{\mu v} + A g_{\mu v} = 8\pi G T_{\mu v} \qquad (1)$$

The constant Λ was called the cosmological constant. It was quickly abandoned after the discovery that the universe is expanding. The Λ term would correspond to the energy of the vacuum – if the ground state of vacuum has a non-vanishing contribution to gravitational stress-energy, it would amount to Λ being non-zero in the above equation. In quantum mechanics, the vacuum can give rise to short-lived, virtual particleantiparticle pairs that can, at least in theory, contribute to Λ. However, simple estimates of the contribution of the various known fields to Λ result in outrageously high values that would have caused the universe to rapidly re-collapse after the Big Bang. Since no viable theory of quantum gravity exists at present, there exists no reliable way to calculate the effects of quantum vacuum states on gravitational phenomena.

A cosmological constant term corresponds to an equation of state with *w* = –1. Other forms of dark energy are also conceivable, for instance arising from scalar fields. In particular, a scalar field whose equation of state approaches that of the cosmological constant term is thought to have been responsible for inflation. It is readily shown from the Friedmann equation that a universe in which the dominant energy is the cosmological constant will increase exponentially in size, which allows for inflation as long as the universe remains dominated by the field.

In the early 1990's, the cosmological constant term was revived as type 1A supernova observations indicated that the universe is in a period of accelerated expansion. This is impossible if the universe is dominated by matter, radiation, curvature or any form of energy with w$\geq$ –1/3. The standard model of cosmology was revised to include a cosmological constant term that contributes to the total energy density:

$$\Omega_{\Lambda} = \left(8\pi G / 3H^{2}\right)\rho_{\Lambda} \qquad (2)$$

Recent observations support a cosmological model with $\Omega_A \approx$ 0.7, $\Omega_M \approx$ 0.3 and $\Omega \approx$1.0. These parameters imply that the universe has zero (or vanishingly small) curvature, and that dark energy is currently the strongest driving force in the universe's evolution. Since the density of dark energy does not decrease with the scale factor, it is expected that in time it will dominate the universe and give rise to a period of exponential expansion until all unbound structures fall outside each other's horizon (unless something happens to end the domination of dark energy).

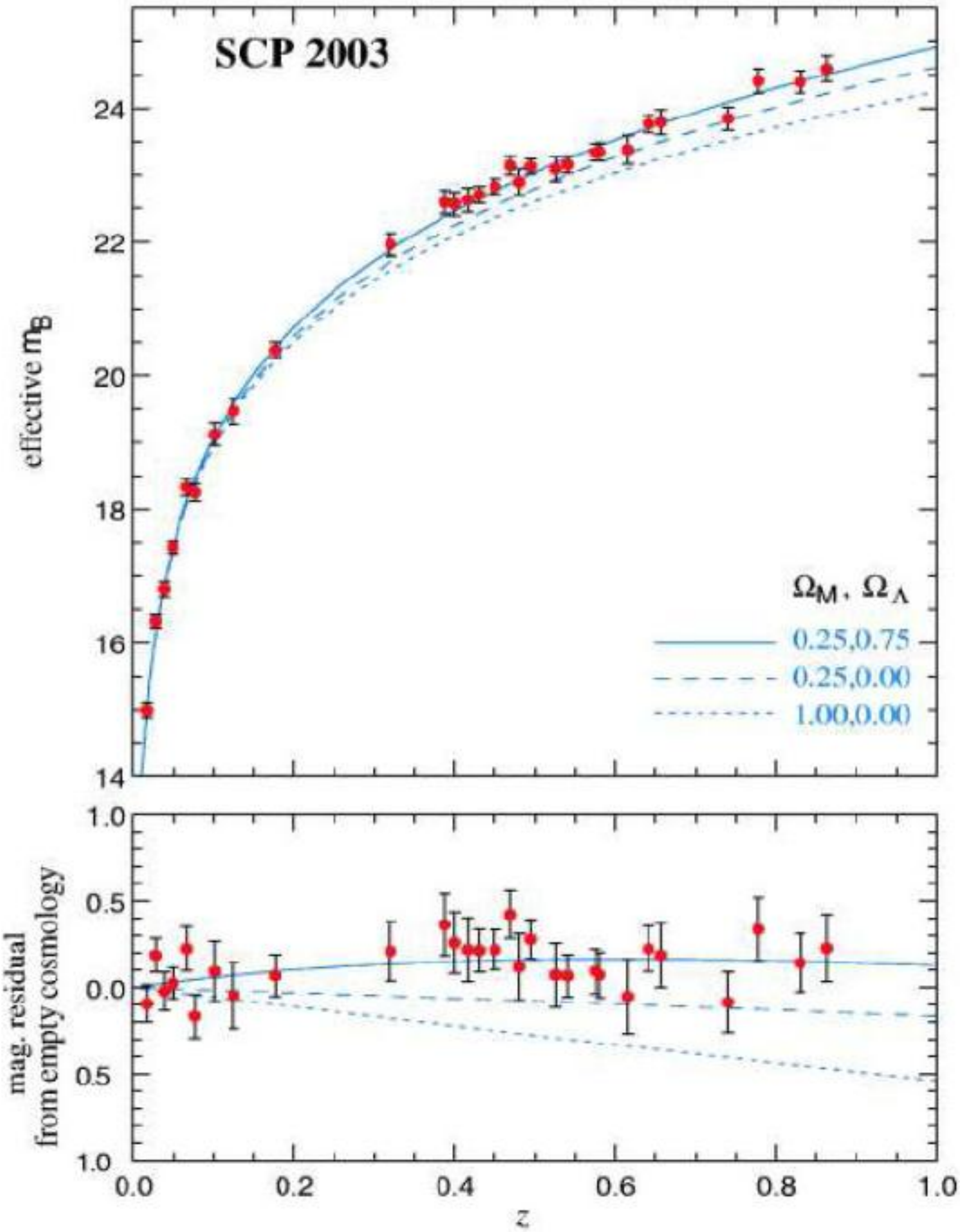

Figure 2. Hubble diagram for supernovae indicating an accelerating universe. Reproduced from [3]

II. Suggested Features

Although the Λ-CDM model that incorporates both dark matter and dark energy is highly successful at explaining features of the observed Universe, it suffers from the lack of insight into the nature of dark matter and dark energy. Given that, it is reasonable to attempt to formulate models that do not require those features. One class of leading alternative models postulates that the general theory of relativity is only approximately correct. In other words, Equation (1) above for the metric holds only approximately. It then becomes imperative to find 'the' equation of motion for the metric. Any such equation must of course reduce to (1) in the domains where it has been tested to high accuracy, such as the Solar System. Ideally, such an equation should also predict that a homogeneous, isotropic Universe can end up in a phase of accelerated expansion either at late times (providing an alternative for dark energy), or at very early times (since this would provide a mechanism for inflation, which is a leading mechanism for solving other puzzles about the observed Universe). Alternatively, in such a model gravity might deviate sufficiently from GR on galactic scales to explain the observed rotation curves without invoking dark matter.

The need for a more general equation for the metric also arises from attempts at unifying gravity with quantum mechanics. In most such models, higher order terms must enter the gravity Lagrangian, and hence modify the equation of motion corresponding to gravity – whether it is quantized (gravitons) or simply remains a description of the underlying space-time, but now quantum fields are treated in curved space-times. The model proposed in [4] generalizes GR by modifying the Einstein-Hilbert action. In its normal form, the metric part of the Lagrangian reads:

$$S[g] = \int R\sqrt{-g}\,d^4x \tag{3}$$

In equation (3), g is the determinant of the metric and R is the Ricci scalar. The simplest generalization that can be made is to write

$$S[g] = \int (R)\sqrt{-g}\,d^4x \tag{4}$$

Equation (4) introduces some function of the Ricci scalar f(R). One might imagine that more general replacements for the metric action could depend, for instance, on derivatives of R. However, there is a known 'no-go' result in classical mechanics due to Ostrogradski that disallows such theories since they are found to introduce instability in the equations of motion if derivatives of higher order than two appear.

The presence of f(R) can be shown to modify the equation for the metric (1) as follows (dropping the cosmological constant term):

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}Rg\alpha\beta = T^{curv}_{\alpha\beta} + \frac{T^{M}_{\alpha\beta}}{f'(R)} \tag{5}$$

In (5), the usual matter stress-energy tensor is denoted as $T^{M}_{\alpha\beta}$. An entirely new $T^{curv}_{\alpha\beta}$ (call it the curvature stress-energy) appears from the higher order effects that can contribute to the Einstein tensor even in the absence of matter stress-energy. It can be shown that

$$T^{curv}_{\alpha\beta} = \frac{1}{f'(R)}\left\{\frac{1}{2}g\alpha\beta\left[f(R) - Rf'(R)\right] + f'(R)^{;\mu\nu}\left(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu}\right)\right\} \tag{6}$$

Clearly the case f(R) = R should recover equation (1), and it is easy to see that it does. In this case, f'(R) = 1 so the coefficient of $T^{M}_{\alpha\beta}$ is what it should be. Meanwhile, the curvature stress-energy disappears, since the combination f (R) −Rf'(R) vanishes and the second term cancels out.

This model was originally intended to replace dark energy. Treating the higher order terms as a source of effective curvature contained within $T^{curv}_{\alpha\beta}$ allows one to keep using the Friedmann equations to describe the evolution of the Universe, with the extra sources from $T^{curv}_{\alpha\beta}$. In particular, the equation for the scale factor is

$$\ddot{a}/a = -\frac{1}{6}\rho_{total} + 3p_{total} \tag{7}$$

Assuming that the universe is currently matter-dominated, the quantities in equation (7) can be decomposed as follows: $\rho_{total} = \rho_{curv}$ + ρ_M and $p_{total} = p_{curv}$ (since non-relativistic matter has negligible pressure). It can also be shown that

$$\rho_{curv} = \frac{1}{f'(R)}\left\{\frac{1}{2}\left[f(R) - Rf'(R)\right] - 3H\dot{R}f''(R)\right\} \tag{8}$$

and that the 'equation of state' for the curvature tensor is

$$w_{curv} = -1 + \frac{\ddot{R}f''(R) + \dot{R}\left[\dot{R}f'''(R) - Hf''(R)\right]}{\left[f(R) - Rf'(R)\right]/2 - 3H\dot{R}f''(R)} \tag{9}$$

Again, equation (8) reduces to GR for f(R) = R, in which case f(R) −Rf '(R) vanishes and f''(R) = 0, so that $\rho_{curv} = 0$. It is not clear what happens to the equation of state (the fraction is the indeterminate form 0/0 for f(R) = R), but since the energy density vanishes, the pressure also vanishes for any finite value of w.

Given this model, it is possible (at least in theory) to determine f(R) by working backwards. The Friedmann equation (7) or its first-order equivalent can be manipulated into an equation for f(R(z)), where z is the redshift. The cosmological data for H(z) can then be used to determine f(R). However, the model studied in [4] does not attempt to do so. It assumes a simple form of f(R) as follows:

$$f(R) = f_0 R'' \tag{10}$$

Here, GR is recovered in the limit n = 1. This model can in fact be successful in matching the SNIa data [5] and the estimated age of the Universe for a range 1.366 < n < 1.376.

Interestingly enough, the model may also serve to explain galactic rotation curves without the use of dark matter. By solving for the Schwarzschildlike metric in this model and taking the appropriate classical limit, the gravitational potential outside a spherically symmetric mass distribution is found to be

$$\Phi(R) = -\frac{GM}{r}\left[1+\left(\frac{r}{r_c}\right)^{\beta}\right] \qquad (11)$$

where $\beta = \beta(n)$ is given by a fairly complicated relationship

$$\beta = \frac{12n^2 - 7n - 1 - \sqrt{36n^4 + 12n^3 - 83n^2 + 50n + 1}}{bn^2 + 4n - 2} \qquad (12)$$

It follows immediately from (12) that the n = 1 limit corresponds to $\beta = 0$, which recovers the typical r^{-1} form of the classical Newtonian potential. The rotation curve may then be evaluated with standard methods, yielding

$$v_c^2(r) = \frac{GM(r)}{r}\left[1+(1-\beta)\left(\frac{r}{r_c}\right)^{\beta}\right] \qquad (13)$$

Here M(r) denotes the total mass contained within the galaxy up to distance r away from its center, assuming the galaxy has some sort of spherical or disk symmetry. r_c is a free parameter of the theory, and corresponds roughly to the scale at which deviations from GR become important. This formula predicts that the rotation curve approaches zero asymptotically at large r, even though observations currently show the rotation curve to be flat towards the edge of galaxies. However, since one necessarily probes only a finite range of r, that finding does not automatically discount the model, and in fact is it possible to find fairly good agreement between this model and observations.

Figure 3 shows some sample plots of theoretical rotation curves of LSB galaxies where the values of r_c and β have been fitted for, based on available data. The authors show 15 such fits; only nine of which have been reproduced here. Of the 15 galaxies considered, ten show good to excellent agreement (such as the middle plot in figure 3), and only three are unsatisfactory (such as the lower left plot in figure 3).

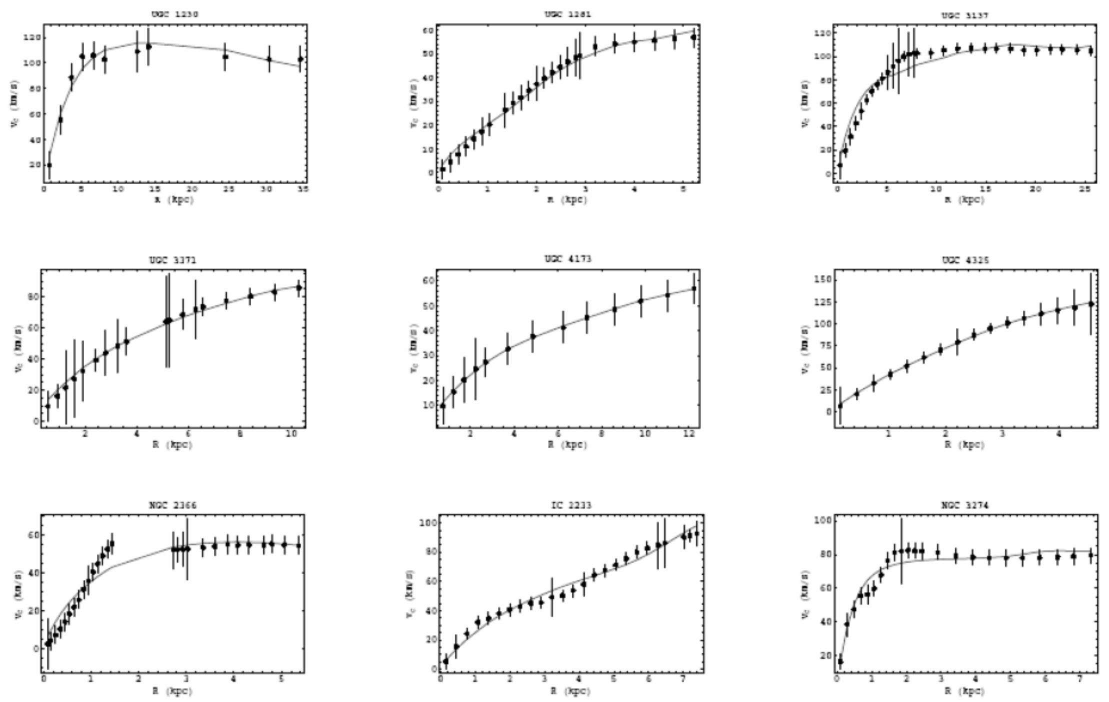

Figure 3 : Best-fit rotation curves for a sampling of galaxies. Reproduced from [6]

The best-fit range is β = 0.58 ± 0.15, corresponding to a range 1.34 < *n* < 2.41. This range agrees with the result obtained from the best-fit of accelerated expansion at the lower end of the scale. Therefore, it is possible to make the claim (actually made in [4]) that this opens up the possibility to dispense with the invisible energy content of the universe currently required for standard cosmology by introducing this alteration to GR encompassed in equations (4) and (10). It is in fact entirely plausible that equation (10) may not be the correct model for f(R) gravity and that the correct model for gravity is more complicated – or possibly simpler; at any rate using an irrational value for the exponent n without some theoretical justification seems unappealing. Nonetheless, the success of a simple model at explaining astrophysical observations warrants further research into such models in the hope that one may be found that makes a full account for the invisible energy content.

III. Explanation and Finding

The model proposed in this paper seems to be at least as viable as leading dark energy models in providing a mechanism for the observed accelerated expansion of the Universe. The authors of [5] claim that a simple Λ term (vacuum energy, w = –1) is "ruled out" by the spectacular failure of simple models to calculate its value (higher than the experimental value by a factor of 10^{30} in energy scale), although this failure probably indicates simply the failure of the simple models per se and a possibility for new physics. However, that failure does little in terms of compelling evidence for the Λ model, and the model presented in [4] remains a viable alternative. Another class of models for dark energy, the so-called "quintessence" models, introduce dark energy as a dynamical field whose equation of state is close to, but not exactly, w=–1. The theory for such models is fairly similar to that of inflationary models, since a similar effect is sought after, simply at a different energy scale. There is little experimental evidence to decide the issue, since the equation of state for dark energy is not very well constrained by existing models, although experiments are underway to measure it more accurately. In any event, quintessence models can explain the observations, but there are no *a priori* choices for the interaction potentials of the fields from fundamental quantum field theory, and there is little reason to prefer such models.

When it comes to replacing dark matter, however, this model runs into serious difficulties not encountered in the standard cold dark matter analysis. Granted, the model can match the rotation curves of galaxies. Furthermore, the best-fit range of n for rotation curves is also consistent with the best-fit range of n from accelerated expansion. Dark matter, however, also plays an important role in the early universe in structure formation. This model does not present a viable alternative in this regard. Recall that the dark matter was able to collapse gravitationally in the early universe without generating a restorative force from the plasma because it was otherwise decoupled from it. In this model, the gravitational interaction between baryons is modified, and may generate additional attraction between matter on galactic scales. However, the collapse of baryons would result in a stronger counter from the radiation pressure associated with the plasma coupled to the baryons, and this would very likely prevent the formation of structure even with the additional attraction – or at the very least inhibit it more than the experimental data allows. Furthermore, the difference in the oscillations of the baryonic matter in this model would become encoded in the CMB acoustic peaks. At present, however, the CMB data from WMAP strongly favors the cold dark matter hypothesis [7], claiming that models without cold dark matter of any kind are a "very poor" fit to the spectrum. The relevant data is shown in figure 4. Although the fit is made for a standard cosmological model, the constraints on Ω_M can be made independently of w if a flat universe is still assumed. If that were the case, the agreement between the 'dark matter' and 'dark energy' ranges of the exponent n would in fact lessen the value of the model, since in order to provide the 'dark energy' one also needs to introduce an unacceptable galacticscale component that acts like dark matter (over and above the presence of regular cold dark matter), which renders the model invalid.

Unfortunately, it is beyond my ability to fully determine how the f(R) model would affect the CMB spectrum (as opposed to dark energy), and so I cannot tell whether it agrees with the rest of the CMB data. However, in principle there is a way that the f(R) model can be compared to dark matter models, if one could find two galaxies that, for instance, orbit around their center of mass, or are in the process of merging. In the dark matter model, the dark matter content of each galaxy could be determined from the rotation curve about their individual centers; however, one expects that the dark matter is entirely confined to the individual galaxies, so that the gravitational attraction between the two galaxies may be entirely determined from the dark matter content. The f(R) model, however, predicts that the gravitational interaction between the galaxies would be different from standard GR, so that the motion around their center of mass would differ from the GR+ dark matter prediction. This difference could possibly be detected if the conditions are favorable.

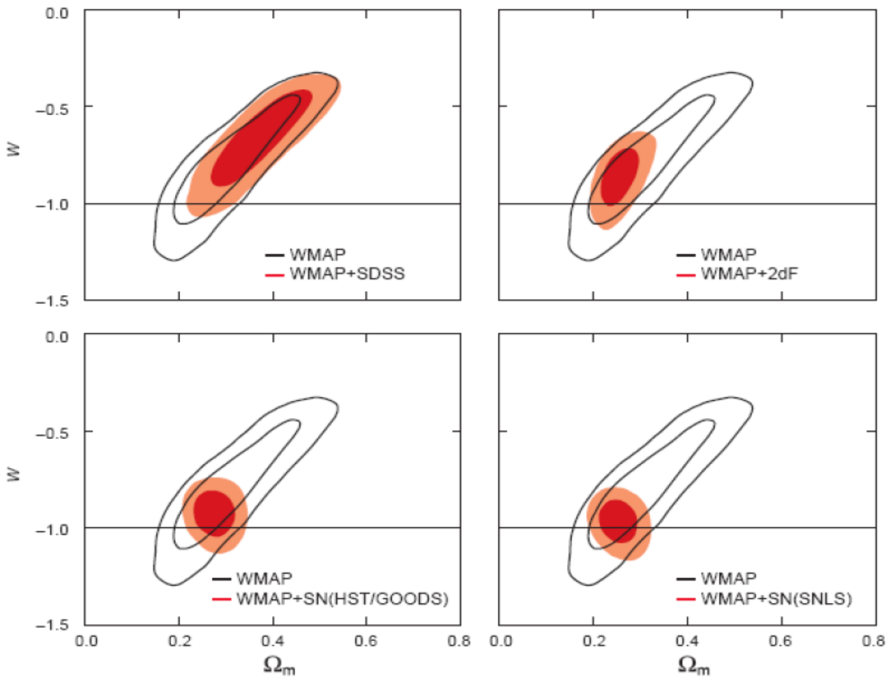

Figure 4. Constraints on *w* and _M from WMAP.
Reproduced from [7]

IV. Acknowledgement

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